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Superconducting correlation in the one-dimensional t-J model with anisotropic spin interaction

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Abstract. A variant of the one-dimensional t-J model with anisotropic spin interaction is studied by the nested algebraic Bethe ansatz method. The gapless charge excitations and the gapful spin excitations are obtained. It is shown that the singlet-superconducting correlation dominates in the low-density region by applying the finite-size scaling analysis in the conformal field theory.

In these years, strongly correlated electron systems have drawn much attention. It is partly caused by the discovery of the copper–oxide high-T_c superconductors. The one-dimensional strongly correlated electron systems play an important role in the study of them. It is partly because some of them can be solved exactly and the non-perturbative results thus obtained contain some of the essential properties of the strongly correlated systems.

Besides the Hubbard model [1–3], the t-J model [4] is regarded as one of the most basic models which contains the essence of strong correlation. The Hamiltonian is

$$\mathcal{H}_{tJ} = \sum_{\langle i,j\rangle} \left[-t \sum_{\sigma=\uparrow,\downarrow} \mathcal{P}(c^\dagger_{i\sigma} c_{j\sigma} + c^\dagger_{j\sigma} c_{i\sigma})\mathcal{P} + J\left(S^x_i S^x_j + S^y_i S^y_j + S^z_i S^z_j - \frac{1}{4} n_i n_j \right)\right]$$

where n's are the number operators given by $n_i = n_{i\uparrow} + n_{i\downarrow} = c^\dagger_{i\uparrow} c_{i\uparrow} + c^\dagger_{i\downarrow} c_{i\downarrow}$ and the spin operators are $S^k_i = \frac{1}{2}\sum_{\alpha,\beta} c^\dagger_{i\alpha}\sigma^k_{\alpha,\beta} c_{i\beta}$ with the usual Pauli matrices σ's. The Gutzwiller projector $\mathcal{P} = \prod_{j=1}^{L}(1 - n_{j\uparrow} n_{j\downarrow})$ restricts the Hilbert space by forbidding double occupancies and hence represents strong correlation. In one dimension, the exact solution was obtained for the supersymmetric case ($2t = J$) [5–8] using the Bethe ansatz method. The long-distance behaviour of the correlation functions was also investigated by applying the finite-size scaling analysis in the conformal field theory to the excitation spectra obtained by the Bethe ansatz method [9]. It was shown that the superconducting correlation does not exceed the others such as the spin-density wave (SDW) and the charge-density wave (CDW) correlations at any filling. However, the region where the superconducting correlation is dominant was found between the low-density supersymmetric region and the 'phase-separated' region ($J \gg t$) by numerical diagonalization for finite clusters [10]. Since the numerical results have ambiguity due to the finite-size effect, it is highly desirable to have exact results.

In this paper, we study a one-parameter family of one-dimensional correlated electron systems which includes the ordinary supersymmetric t-J model. The Hamiltonian is

$$\mathcal{H}_{tJ\gamma}=\sum_i\Bigg[-\sum_\sigma\mathcal{P}(c^\dagger_{i\sigma}c_{i+1\sigma}+c^\dagger_{i+1\sigma}c_{i\sigma})\mathcal{P}+2\left(S^x_iS^x_{i+1}+S^y_iS^y_{i+1}+\Delta S^z_iS^z_{i+1}-\frac{\Delta}{4}n_in_{i+1}\right)$$
$$+\eta(S^z_in_{i+1}-n_iS^z_{i+1})+2\Delta n_i\Bigg] \qquad (1)$$

where $\Delta^2-\eta^2=1$ and we parametrize them as $\Delta=\cosh\gamma$ and $\eta=\sinh\gamma(\gamma\in\mathbb{R}_{\geqslant 0})$†. When $\gamma=0$, our Hamiltonian reduces to the ordinary supersymmetric t-J model. When the number of electrons coincides with the number of lattice sites, our model becomes the $s=\frac{1}{2}$ XXZ spin chain. The Hamiltonian (1) commutes with the transfer matrix of the solvable two-dimensional classical lattice model associated with the supersymmetric quantum affine superalgebra $U_q(\widehat{sl(2|1)})(q=\mathrm{e}^\gamma)$, which is a special case of the model given by Perk and Schultz [11, 12]. Thus, one could say that the Hamiltonian is 'q-supersymmetric'. The ground state, the excitations and the correlation functions are obtained by the nested algebraic Bethe ansatz method, the finite-size scaling analysis in the conformal field theory and the numerical diagonalization for small clusters. We find that there is always a finite excitation gap in the spin sector ('*spin gap*'), and the superconducting correlation dominates in the low-density region in contrast to the ordinary supersymmetric t-J model. In our study, we encounter the problem of finding suitable distributions of the roots of the Bethe ansatz equation which correspond to the low-energy excitations. For the usual supersymmetric t-J model, an ansatz was proposed for the problem and the validity was confirmed by comparing the results with those from numerical studies [5–8]. An ansatz is also proposed in our case and the validity is established by the help of the numerical-diagonalization technique.

The third term of the Hamiltonian (1) breaks the parity invariance. This parity-breaking term is necessary for the sake of the integrability. The relation between the spin gap and the parity-breaking term is clarified in our study. It is investigated by diagonalizing the system without the parity-breaking term numerically. It is confirmed that the spin gap exists even for the system without the parity-breaking term. This means that the spin gap is not a special feature of the exactly solvable case but it may happen generally for non-integrable cases.

This article is organized as follows. In section 1, the Bethe ansatz equations are given. The ground state is obtained in section 2, the charge and the spin excitations are studied in section 3 and 4. The critical exponents of correlation functions are discussed in section 5.

After finishing this work, the authors noticed the works by Bariev *et al* [23–25] in which the critical exponents of the anisotropic t-J model were obtained by the Bethe ansatz. They used the same hypothesis as ours in solving the Bethe ansatz equations. In their papers, the validity of the hypothesis is not discussed. In this work, however, we have confirmed this hypothesis by using the numerical-diagonalization technique. Moreover, the study of the Hamiltonian without the parity-breaking term is not discussed in their works, which cannot be done by the Bethe ansatz technique. We investigated this problem by numerical diagonalization for small clusters.

† The Hamiltonian (1) can be transformed to that for the doped XY-model with broken parity: $\mathcal{H}_{XY}=\sum_i[-\sum_\sigma\mathcal{P}(c^\dagger_{i\sigma}c_{i+1\sigma}+c^\dagger_{i+1\sigma}c_{i\sigma})\mathcal{P}+2(S^x_iS^x_{i+1}+S^y_iS^y_{i+1})-(\mathrm{e}^\gamma n_{i\uparrow}n_{i+1\downarrow}+\mathrm{e}^{-\gamma}n_{i\downarrow}n_{i+1\uparrow})]$.

1. Bethe ansatz equations (BAEs)

The diagonalization of the Hamiltonian (1) with periodic boundary condition reduces to solving the coupled algebraic equations (BAEs) derived by the nested algebraic Bethe ansatz technique. They are

$$\left(\frac{\sin(p_j+\frac{\mathrm{i}}{2}\gamma)}{\sin(p_j-\frac{\mathrm{i}}{2}\gamma)}\right)^L=(-1)^N\prod_{\beta=1}^{M}\frac{\sin(p_j-\Lambda_\beta+\frac{\mathrm{i}}{2}\gamma)}{\sin(p_j-\Lambda_\beta-\frac{\mathrm{i}}{2}\gamma)}\qquad j=1,2,\ldots,N \tag{2}$$

$$\prod_{j=1}^{N}\frac{\sin(\Lambda_\alpha-p_j+\frac{\mathrm{i}}{2}\gamma)}{\sin(\Lambda_\alpha-p_j-\frac{\mathrm{i}}{2}\gamma)}=-\prod_{\beta=1}^{M}\frac{\sin(\Lambda_\alpha-\Lambda_\beta+\mathrm{i}\gamma)}{\sin(\Lambda_\alpha-\Lambda_\beta-\mathrm{i}\gamma)}\qquad \alpha=1,2,\ldots,M \tag{3}$$

where L is the number of lattice sites, N is the number of electrons, M is the number of down-electrons (magnons), p's are the quasi-momenta of electrons and Λ's are the magnon rapidities [13].

Hereafter we take the following ansatz for the ground state and the elementary excitations: *Λ's are one-strings $\{\Lambda_\alpha\in\mathbb{R}|\alpha=1,\ldots,M\}$ and p's consist of the one-strings $\{p_j=u_j\in\mathbb{R}|j=1,\ldots,N-2M\}$ and the two-strings $\{p_\alpha^\pm=\Lambda_\alpha\pm\mathrm{i}\gamma/2|\alpha=1,\ldots,M\}$.* Note that the real parts of the two-strings coincide with the magnon rapidities and, if $M=N/2$, there are no degrees of freedom for one strings of the quasi-momenta. This is essentially the same ansatz established in [6–8]. In our case, however, Λ's and u's are in the interval $[-\frac{\pi}{2},\frac{\pi}{2}]$ due to the periodicity of the BAEs (2), (3). By taking the logarithm of the BAEs, we have

$$L\phi(u_j,\frac{\gamma}{2})=2\pi\mathrm{i}I_j+\sum_{\beta=1}^{M}\phi\left(u_j-\Lambda_\beta,\frac{\gamma}{2}\right)\qquad j=1,\ldots,N-2M \tag{4}$$

$$L\phi(\Lambda_\alpha,\gamma)=2\pi\mathrm{i}J_\alpha+\sum_{j=1}^{N-2M}\phi\left(\Lambda_\alpha-u_j,\frac{\gamma}{2}\right)+\sum_{\beta=1}^{M}\phi(\Lambda_\alpha-\Lambda_\beta,\gamma)\qquad \alpha=1,\ldots,M \tag{5}$$

where $\phi(z,\alpha)\equiv\log\frac{\sin(z+\mathrm{i}\alpha)}{\sin(z-\mathrm{i}\alpha)}$ †, and $\{I_j|j=1,2,\ldots,N-2M\}$ is a set of integers (or half-odd integers) if M is even (or odd) and the set $\{J_\alpha|\alpha=1,2,\ldots,M\}$ is a set of integers (or half-odd integers) if $N+M+1$ is even (or odd). We order the quantum numbers I's and J's according to $I_j>I_{j+1}$ and $J_\alpha>J_{\alpha+1}$.

In the thermodynamic limit $L,N\to\infty$, the distributions of Λ's can be described by the continuous density given by $L\rho(\Lambda_\alpha)=\lim_{L,N\to\infty}1/(\Lambda_{\alpha+1}-\Lambda_\alpha)$. The energy and momentum up to O(1) are

$$E=\mathrm{i}\sinh\gamma\sum_{j=1}^{N}\phi'\left(p_j,\frac{\gamma}{2}\right)=\mathrm{i}\sinh\gamma\left(L\int\mathrm{d}\Lambda'\,\rho(\Lambda')\phi'(\Lambda',\gamma)+\sum_{j=1}^{N-2M}\phi'\left(u_j,\frac{\gamma}{2}\right)\right) \tag{6}$$

and

$$P=\mathrm{i}\sum_{j=1}^{N}\phi\left(p_j,\frac{\gamma}{2}\right)=\mathrm{i}\;L\int\mathrm{d}\Lambda'\,\rho(\Lambda')\phi(\Lambda',\gamma)+\mathrm{i}\sum_{j=1}^{N-2M}\phi\left(u_j,\frac{\gamma}{2}\right). \tag{7}$$

The integral intervals in (6) and (7) will be discussed in the following sections.

† The branch of the logarithm is fixed by the requirements that $\phi(0,\alpha)=\mathrm{i}\pi$, and $\mathrm{Im}\,\phi(x,\alpha)$ is a continuous monotonic decreasing function in $-\pi/2<x<\pi/2$.

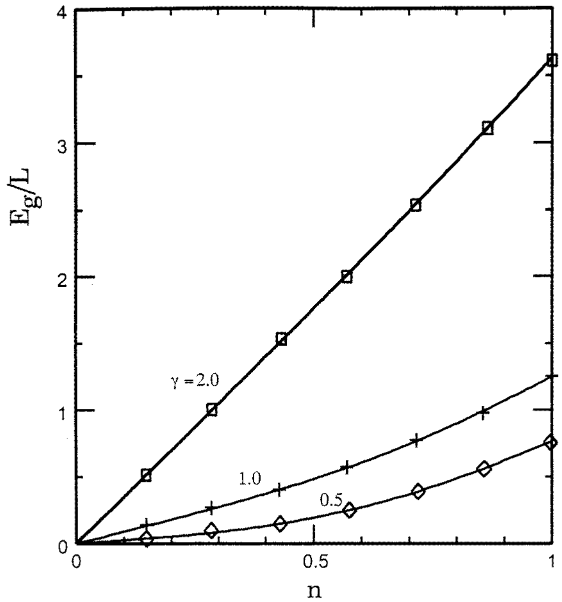

Figure 1. Ground-state energy per site as a function of electron density n. The full lines are obtained by solving (8). We also performed direct numerical diagonalizations of the Hamiltonian for $L = 14$ with $\gamma = 2.0$, 1.0 and 0.5. The results are plotted by $\square$, $+$ and $\Diamond$ respectively.

2. Ground state

We set N to be even for simplicity. Since the spin interaction is anti-ferromagnetic, the total S^z for the ground state is expected to be zero. This can be achieved by setting $M = N/2$, i.e. for the sector without one-strings $p_j = u_j$. We also require that the momentum P to be zero. We propose the following ansatz for J's: the distributions of J's for the ground state is restricted as $J_{\max} \geqslant |J_\alpha| \geqslant J_{\min}$, where $J_{\max} = \frac{L-M-1}{2}$ and $J_{\min} = \frac{L-2M+1}{2}$.

In the thermodynamic limit, we assume that Λ's are distributed only in the regions $[-\pi/2, -Q_g]$ and $[Q_g, \pi/2]$ in accordance with the distribution of the quantum numbers J's. BAEs (4) and (5) are reduced to

$$2\pi \mathrm{i}\rho_g(\Lambda) = -\phi'(\Lambda,\gamma) + \left[\int_{-\pi/2}^{-Q_g} + \int_{Q_g}^{\pi/2}\right] \mathrm{d}\Lambda'\,\rho_g(\Lambda')\phi'(\Lambda-\Lambda',\gamma) \qquad (8)$$

where Q_g is determined by

$$\left[\int_{-\pi/2}^{-Q_g} + \int_{Q_g}^{\pi/2}\right] \mathrm{d}\Lambda\,\rho_g(\Lambda) = \frac{N/2}{L} \equiv \frac{n}{2}.$$

The ground-state energy E_g is given by

$$E_g = \mathrm{i}L\sinh\gamma \left[\int_{-\pi/2}^{-Q_g} + \int_{Q_g}^{\pi/2}\right] \mathrm{d}\Lambda\,\rho_g(\Lambda)\phi'(\Lambda,\gamma). \qquad (9)$$

These equations can be solved numerically for arbitrary filling n. The results are given in figure 1.

3. Charge excitations

The charge excitations are those caused by the replacements of the Λ's while keeping $M = N/2$ namely S^z remains to be zero. Thus the elementary excitations for the charge sector consists in making a jump (hole) at the point J_{α_h} and putting a quantum number J_{α_p} at a previously unoccupied region [7, 8]. In the thermodynamic limit, BAEs (4) and (5) are reduced to

$$2\pi i\rho_c(\Lambda) = -\phi'(\Lambda, \gamma) - \frac{2\pi \mathrm{i}}{L}\delta(\Lambda - \Lambda_h) + \frac{1}{L}\phi'(\Lambda - \Lambda_p, \gamma) + \left[\int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2}\right] \mathrm{d}\Lambda'\, \rho_c(\Lambda')\phi'(\Lambda - \Lambda', \gamma) \tag{10}$$

retaining terms up to $\mathrm{O}(L^{-1})$, where Λ_p and Λ_h denote the position of the hole and particle in the sea of two strings associated with the quantum number J_{α_h} and J_{α_p} respectively. Q_c is determined by

$$\left[\int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2}\right] \mathrm{d}\Lambda\, \rho_c(\Lambda) = \frac{(N-2)/2}{L}.$$

For convenience, we decompose $\rho_c(\Lambda)$ into the regular part and the singular part as $\rho_c(\Lambda) = \rho_{c0}(\Lambda) - \frac{1}{L}\rho_{c1}(\Lambda) - \frac{1}{L}\delta(\Lambda - \Lambda_h)$, where $\rho_{c0}(\Lambda)$ satisfies

$$2\pi \mathrm{i}\rho_{c0}(\Lambda) = -\phi'(\Lambda, \gamma) + \left[\int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2}\right] \mathrm{d}\Lambda'\, \rho_{c0}(\Lambda')\phi'(\Lambda - \Lambda', \gamma). \tag{11}$$

For $\rho_{c1}(\Lambda)$, we have

$$2\pi \mathrm{i}\rho_{c1}(\Lambda) = \phi'(\Lambda - \Lambda_h, \gamma) - \phi'(\Lambda - \Lambda_p, \gamma) + \left[\int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2}\right] \mathrm{d}\Lambda'\, \rho_{c1}(\Lambda')\phi'(\Lambda - \Lambda', \gamma). \tag{12}$$

The excitation energy ΔE from the ground state and the momentum P are given by

$$\Delta E = \mathrm{i}\sinh\gamma\left(\phi'(\Lambda_p, \gamma) - \phi'(\Lambda_h, \gamma) - \left[\int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2}\right] \mathrm{d}\Lambda\, \rho_{c1}(\Lambda)\phi'(\Lambda, \gamma)\right) \tag{13}$$

$$P = \mathrm{i}\left(\phi(\Lambda_p, \gamma) - \phi(\Lambda_h, \gamma) - \left[\int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2}\right] \mathrm{d}\Lambda\, \rho_{c1}(\Lambda)\phi(\Lambda, \gamma)\right). \tag{14}$$

Solving (12) numerically, the dispersion relation for the elementary charge excitations was obtained. The results for $\gamma = 2$ and $n = 0.45$ are shown in figure 2. The results for other parameters do not change in an essential manner, namely *the charge excitation is always gapless*.

4. Spin excitations

The spin excitations can be considered as excitations coming from destroying the two-strings $p^{\pm}$'s and creating one-strings u's. To study the elementary ones, let us consider the case of $M = N/2 - 1$ magnons. We assume that, in the sea of the quantum numbers J's there are no jumps [7, 8]. Then, in the thermodynamic limit, BAE (5) becomes

$$2\pi \mathrm{i}\rho_s(\Lambda) = -\phi'(\Lambda, \gamma) + \frac{1}{L}\phi'\left(\Lambda - u_1, \frac{\gamma}{2}\right) + \frac{1}{L}\phi'\left(\Lambda - u_2, \frac{\gamma}{2}\right) + \left[\int_{-\pi/2}^{-Q_s} + \int_{Q_s}^{\pi/2}\right] \mathrm{d}\Lambda'\, \rho_s(\Lambda')\phi'(\Lambda - \Lambda', \gamma) \tag{15}$$

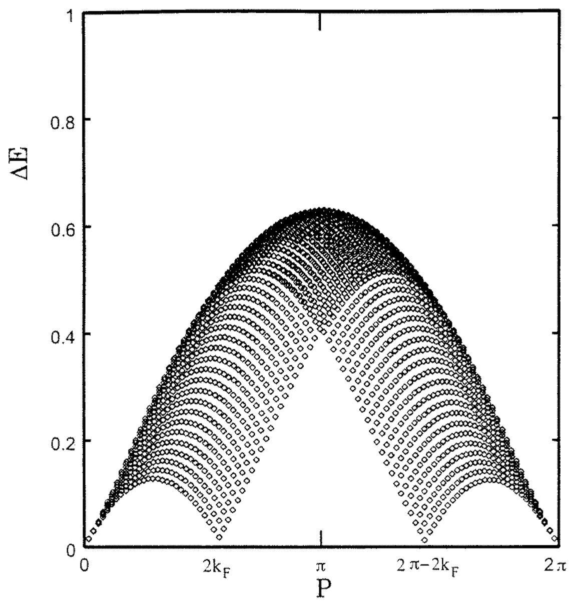

Figure 2. Dispersion of the elementary excitations in the charge sector for $\gamma = 2$ and $n = 0.45$. Sufficiently many points in the continuous spectrum are shown. The momentum P is periodic with period 2π. The gapless points are at $P = 0, 2k_F$ and $2\pi - 2k_F$.

where u_1 and u_2 are one-string quasi-momenta, and Q_s is determined by

$$\left[\int_{-\pi/2}^{-Q_s} + \int_{Q_s}^{\pi/2}\right] \mathrm{d}\Lambda\, \rho_s(\Lambda) = \frac{(N-2)/2}{L}.$$

It is convenient to decompose $\rho_s(\Lambda)$ into contributions of O(1) and O(L^{-1}) as $\rho_s(\Lambda) = \rho_{s0}(\Lambda) - \frac{1}{L}\rho_{s1}(\Lambda)$, where $\rho_{s0}(\Lambda)$ satisfies the same equation as (11) obtained by replacing all the suffixes c to s. Then the integral equation for $\rho_{s1}(\Lambda)$ is obtained as

$$\begin{aligned} 2\pi \mathrm{i}\rho_{s1}(\Lambda) = &-\phi'\left(\Lambda - u_1, \frac{\gamma}{2}\right) - \phi'\left(\Lambda - u_2, \frac{\gamma}{2}\right) \\ &+\left[\int_{-\pi/2}^{-Q_s} + \int_{Q_s}^{\pi/2}\right] \mathrm{d}\Lambda'\, \rho_{s1}(\Lambda')\phi'(\Lambda - \Lambda', \gamma). \end{aligned} \tag{16}$$

The excitation energy ΔE from the ground state and the momentum P are

$$\Delta E = \mathrm{i}\sinh\gamma \left(\phi'\left(u_1, \frac{\gamma}{2}\right) + \phi'\left(u_2, \frac{\gamma}{2}\right) - \left[\int_{-\pi/2}^{-Q_s} + \int_{Q_s}^{\pi/2}\right] \mathrm{d}\Lambda\, \rho_{s1}(\Lambda)\phi'(\Lambda, \gamma)\right) \tag{17}$$

$$P = \mathrm{i}\left(\phi\left(u_1, \frac{\gamma}{2}\right) + \phi\left(u_2, \frac{\gamma}{2}\right) - \left[\int_{-\pi/2}^{-Q_s} + \int_{Q_s}^{\pi/2}\right] \mathrm{d}\Lambda\, \rho_{s1}(\Lambda)\phi(\Lambda, \gamma)\right). \tag{18}$$

Solving (16) numerically, the dispersion relation for the elementary spin excitations was obtained. The results for $\gamma = 2$ and $n = 0.45$ are shown in figure 3. The results for other parameters do not change in an essential manner, namely *the spin excitation is always gapful*. The spin gap as a function of γ is also shown in figure 4 and one can see that the gap increases as holes are doped.

To study the effect of the parity-breaking term, we calculated the spin–spin correlations $\langle S_i^z S_j^z \rangle$ for the parity-unbroken Hamiltonian $\mathcal{H}_{tJ\gamma} - \sum_i \eta(S_i^z n_{i+1} - n_i S_{i+1}^z)$ [15] by a

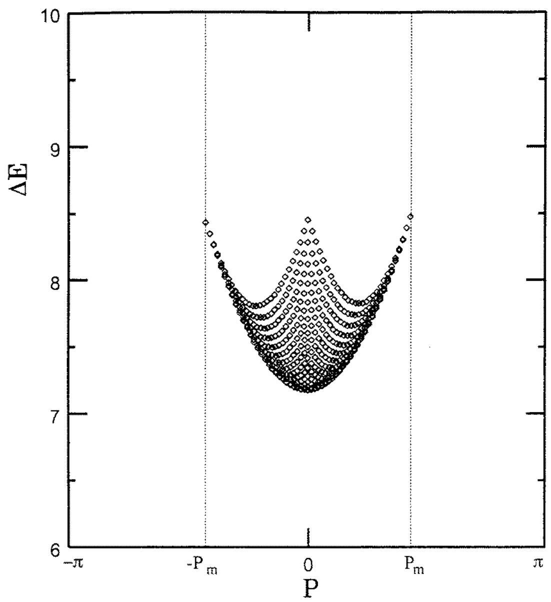

Figure 3. Dispersion of the elementary excitations in the spin sector for $\gamma = 2$ and $n = 0.45$. Sufficiently many points in the continuous spectrum are shown. The momentum P is periodic with period 2π. There is no gapless point. Note that the momentum P for the elementary spin excitation is restricted in $[-P_m, P_m]$, where P_m depends on γ and n.

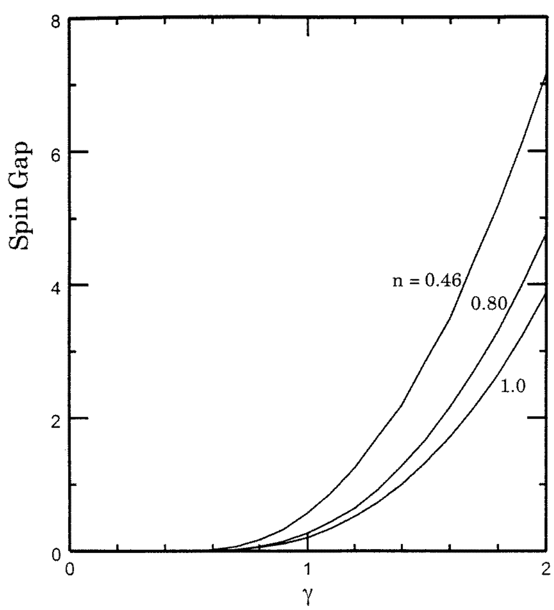

Figure 4. Spin gap as a function of γ. For $n = 1$, the spin gap of our model reduces to that of the XXZ spin chain (see equation (1)) [14].

numerical technique. The results are shown in figure 5 and they indicate that the correlation decays exponentially. Hence, there is still a spin gap. Thus, one of the novel properties

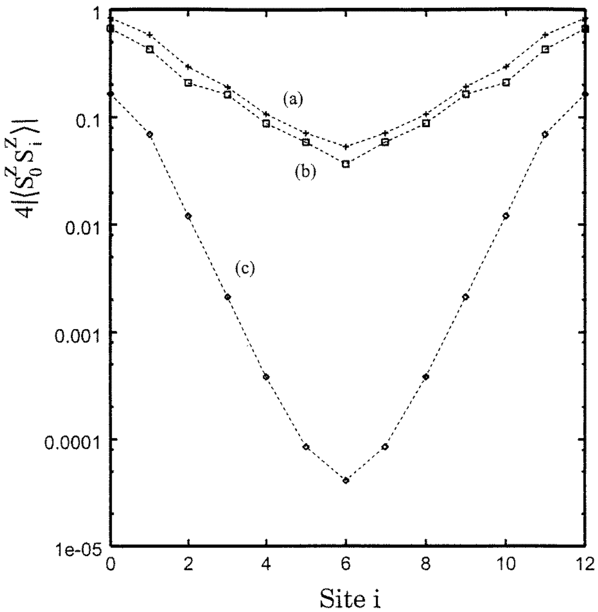

Figure 5. Spin–spin correlations on a logarithmic scale for the parity-unbroken Hamiltonian $\mathcal{H}_{tJ\gamma} - \sum_i \eta(S_i^z n_{i+1} - n_i S_{i+1}^z)$. The results are obtained by numerical diagonalization of $L = 12$ systems for (*a*) $N = 10$, $\gamma = 1.5$, (*b*) $N = 8$, $\gamma = 1.5$ and (*c*) $N = 2$, $\gamma = 2.0$.

in the Hamiltonian (1), *'spin gap'*, is preserved without the parity-breaking term. In other words, the parity-breaking term is *not* relevant for the *spin gap*.

5. Correlation functions

Consider a field-theoretic description of the low-lying excitations. Since the dispersion for the low energy charge sector is approximately linear for $0 < n < 1$, and the gapful spin sector is irrelevant for the low-energy behaviour, we can expect the system can be described by the conformal field theory [16].

Let us consider the excitations described by the density $\rho(\Lambda)$ satisfying

$$2\pi \mathrm{i}\rho(\Lambda) = -\phi'(\Lambda, \gamma) + \left[\int_{-\pi/2}^{Q_-} + \int_{Q_+}^{\pi/2}\right] \mathrm{d}\Lambda'\, \rho(\Lambda')\phi'(\Lambda - \Lambda', \gamma) \qquad (19)$$

and apply the general method of Kawakami–Yang [9] for the finite-size scaling method [17, 18]. Using the Fourier–transform technique, we rewrite (19) as

$$\rho(\Lambda) = 2R_q(2\Lambda) + \int_{Q_-}^{Q_+} \mathrm{d}\Lambda'\, 2R_q(2(\Lambda - \Lambda'))\rho(\Lambda') \qquad (20)$$

where we have introduced the deformed Shiba–function [20] defined by

$$R_q(v) = \frac{1}{2\pi} \sum_{m\in\mathbb{Z}} \frac{\mathrm{e}^{\mathrm{i}mv}}{1 + q^{2|m|}}.$$

The energy is given by

$$E/L = 2\cosh\gamma - 2\pi \sinh\gamma \left[2R_q(0) + \int_{Q_-}^{Q_+} \mathrm{d}\Lambda\, 2R_q(-2\Lambda)\rho(\Lambda)\right]. \qquad (21)$$

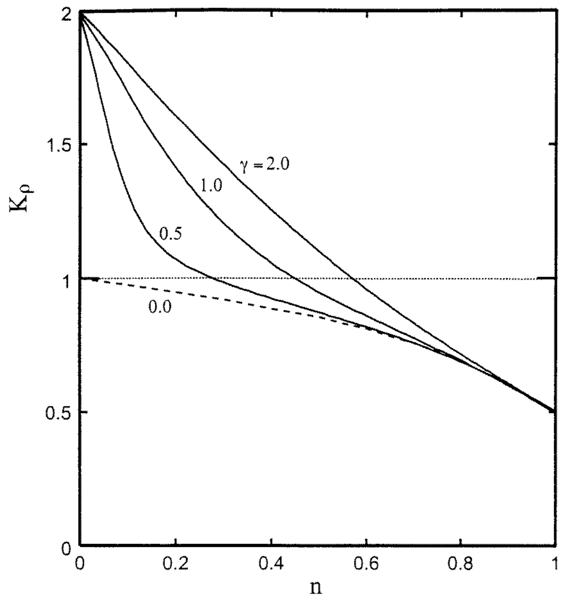

Figure 6. $K_\rho(n)$'s are shown for $\gamma = 0.0, 0.5, 1.0$ and 2.0. The broken line (for $\gamma = 0$ i.e. the ordinary supersymmetric case) denotes the data from [8]. It can be shown analytically that $K_\rho(0) = 2$ and $K_\rho(1) = \frac{1}{2}$ for any $\gamma > 0$.

Table 1. Relation between K_ρ and the critical exponents of the correlation functions.

Correlations	Exponents
$2k_F$ SDW (spin density wave)	Exponential decay
$2k_F$ CDW (charge density wave)	K_ρ
SS (singlet superconductivity)	$1/K_\rho$
TS (triplet superconductivity)	Exponential decay
$4k_F$ CDW (charge density wave)	$4K_\rho$

Thanks to (20) and (21), we can immediately apply the general argument and the results are: (i) the charge sector can be described by the $c = 1$ bosonic conformal field theory i.e. it belongs to the universality class called *the Tomonaga–Luttinger liquid*, (ii) the compactification radius [19] is given by $r = \xi(Q)$, where the *dressed charge* $\xi(\Lambda)$ satisfies $\xi(\Lambda) = 1 + \int_{-Q}^{Q} \mathrm{d}\Lambda'\, 2R_q(2(\Lambda - \Lambda'))\xi(\eta)$, and Q is determined by

$$\left[\int_{-\pi/2}^{-Q} + \int_{Q}^{\pi/2}\right] \mathrm{d}\Lambda\, \rho(\Lambda) = \frac{N/2}{L}.$$

As usual, we parametrize r by $K_\rho = r^2/2$. The equation for ξ was solved numerically and K_ρ as functions of n are shown in figure 6. The relations between K_ρ and the critical exponents are shown in table 1 [21, 22]. As long as $\gamma \neq 0$, the singlet-superconducting correlation is dominant when $K_\rho > 1$ in the low density region (high doping). However, for the usual supersymmetric case ($\gamma = 0$), the superconducting correlation cannot be dominant in any filling [9] as seen in figure 6. This indicates that deformed t-J models including ours may be more appropriate to study superconducting mechanisms than the ordinary t-J model.

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